Thesis subject

Laboratory : Centre de Physique Théorique UMR CNRS 7332

Thesis supervisors : Thierry Martin, Professor at Aix Marseille Université CPT, Aurélien Manchon, professor at Aix Marseille Université (CINAM).

Potential co-supervisors : F. Ronetti (CPT Assistant Professor), B. Grémaud (CPT, CNRS Director of Research).

Title of the thesis subject : "Flying Qbits and entanglement in systems where the quantum Hall effect and superconductivity coexist"

Keywords: quantum Hall effect, graphene, superconductivity, Andreev reflection, crossed Andreev reflection, proximity effect, Matsubara formalism, Keldsyh Schwinger formalism, electron quantum optics;

Description of the thesis subject :

The nanophysics team of CPT has gained over the years international recognition on quantum transport theory in nanoscale systems, when quantum coherence operates at low temperatures. Some of its area of expertise include the description of quantum transport in nanoscale systems (mesoscopic quantum physics or quantum nanophysics) using equilibrium or out of equilibrium formulation of quantum field theory (Keldysh-Schwinger formalism). It is looking for a high level student for this PhD proposal (see candidacy details at the end of this file).

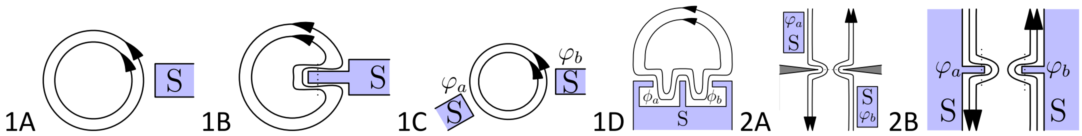
The present text is a PhD topic dealing with the coexistence of edge states in the quantum Hall effect and proximitized superconductivity, looking of novel types of excitations with applications to flying Qbits and quantum information protocols. For this reason, it is a topic connected to quantum entanglement, quantum communication and decoherence of flying Qbits and it is relevant for the present call for PhD fellowships.

In the first part, it concerns the integer quantum Hall effect, but extensions could lead also to the investigation of the fractional quantum Hall regime, which constitutes a novel state of matter where elementary excitations do not resemble electrons: their charge is fractional and their statistics under exchange is intermediate between that of bosons or fermions.

Usually, it is hard to achieve conditions where the (integer) quantum Hall effect coexists with superconductivity. In 2D electron gases from GAAs, the magnetic field kills superconductivity as it exceeds the critical field of the superconductor. However, for graphene 2DEG (which bears a Dirac fermion spectrum close to the Fermi energy), this critical field is much lower and allows type II superconductors to coexist with integer Hall states. This constitutes an excellent testing ground to study crossed Andreev reflection in this special setting, where the existence of edge states is established.

Andreev reflection has occupied both the experimental and theoretical community of mesoscopic physics for several decades (early 1980's), as it allows to study the Josephson effect: when dealing with a junction between two superconductors, a situation where electron and hole closed orbits give rise to so called Andreev bound states, giving rise to the DC and AC Josephson effect.

For a superconductor connected to several normal metal leads, since the mid-nineties, attention has also focused on so-called Cooper pair splitting – also called crossed Andreev reflection, where the two constituent electrons of a Cooper pair propagate in two different normal metal leads as delocalized spin/energy entangled electron states in a condensed matter. This crossed Andreev reflection process is typically accompanied by electron cotunneling, a process where electrons are transferred from one lead to the other via virtual excitation states of the superconductor.



In the present context (superconductivity and IQHE) one has the coexistence of two topological states of matter with very different nature. If one places a superconducting « finger » (whose width is smaller than the coherence length) on top of a 2D graphene or AsGa electron gas in the IQHE, it is interesting to study the effect of crossed Andreev processes between the 2 sides of the finger. In the filling factor nu=2 the two spin polarizations are coded on the edge states and crossed Andreev reflection is possible.

Given the one-dimensional nature of edge states, we will adopt a one-dimensional toy model for the whole system. Position "0" will be associated with the tip of the superconducting finger (which has a length L). The superconducting proximity effect will thus operate for coordinates x in the interval {-L;+L}. Andreev reflection (and cotunneling) will occur at all locations in this interval (within a coherence length). However, crossed Andreev reflection will couple fermion degrees of freedom only across the finger, that is between locations x and -x (within a coherence length) in this interval.

Assuming these physical ingredients (BCS superconductivity, nu=2 edge states with opposite spins, spin preserving electron tunneling between the superconductor and the edge states), a starting point will be to write the full Hamiltonian in Nambu spinor notation. One will then have the choice, for future developments to either study the (closed) system at thermal equilibrium (Mastubara formalism), focusing on a quantum droplet with a superconducting finger, or to study quantum transport in an open system consisting of two opposed spin edge states connected to a source and drain ohmic contact where a voltage bias is applied. In the description of this internship project, we start with system at thermal equilibrium.

The full Hamiltonian will allow to transfer to a functional integral formalism using fermionic coherent states (in terms of Grassmann variables). We will thus generate a Euclidean action which is quadratic in terms of the fermionic degrees of freedom describing both the superconducting degrees of freedom and the edge state fermions.

The first important task will be to integrate out the superconducting degrees of freedom in order to obtain an effective action for the edge state which includes contributions attributed to the proximity effect.

Next, we will integrate the fermionic degrees of freedom of the edge states in order to obtain the partition function of the full system. By taking the logarithmic derivative of the partition function with respect to the flux enclosed in the droplet, we will compute the Josephson current/persistent current of the system. Alternatively, one can compute the linear response function (electric susceptibility) when a small electric field polarizes the charge excitations along the edge of the droplet.

In order to treat the case of an open system, the (real time) action will be derived using the Keldysh formalism of non-equilibrium quantum field theory. The main observable that we will focus on will be the charge current along the edge, both before and after the superconductor. We will first study the case of a DC voltage and observe how the current is modified by crossed Andreev reflection processes. Next, in the context of single electron excitations of so called "Electronic Quantum Optics", we will study the effect of injecting a fermion along the edge (before the superconducting finger) using a prepared state and follow its evolution once this excitation has propagated downstream from the superconductor. Selective population of edge states by electron excitations or voltage drives is also a possibility.

So far, Coulomb interactions have been neglected between the two spin polarized edge states. Tunneling between the two is irrelevant unless spin flip processes are operating. One could, for instance, include Coulomb interactions between the two edge states, using a chiral Luttinger liquid description for the fermion (abelian bosonization). The Coulomb interaction between edge state excitations can be assumed to be short ranged, and it is then quadratic in the bosonic degrees of freedom. It is known to lead to charge fractionalization of the edge states, with a fact and slow mode propagating along the two edge channels (spin charge separation).

Alternatively, one could also focus on the fractional quantum Hall effect in a Laughlin fraction where the system is spin polarized. In the case of Laughlin fractions such as nu=1/3, only the lowest Landau level is partially occupied. Andreev reflection can thus not occur in principle if the superconductor is of pure BCS type. However, magnetic impurities can be the source of spin flip processes which mediate Andreev reflection. Alternatively, unconventional superconductors, which may allow spin flip processes (for instance due to spin orbit coupling) can be required to achieve Andreev reflections. The model and derivation of the action has then to be revisited in this case and one can in principle generate new phenomenon in the context of the fractional Hall effect.

Extension to composite fractions of the fractional quantum Hall effect are possible Extensions include the possibility of searching for parafermions/non abelian topological quantum excitations is such a system or similar ones and to exploit them in quantum information protocols.

This project has both analytical and numerical aspects. Candidates shold have a strong background in Quantum Statistical Physics and some knowledge of zero and finite temperature many body theory, as well as some background in phenomenological (Ginzburg Landau) and microscopic superconductivity (BCS and Bogoliubov formalism). Some strong analytic skill are a must, but some established experience in numerical methods (even such as Mathematica and Python) for light numerics is also a plus (strong background in numerics is even better). Candidates should be ready to learn functional integration techniques using fermion coherent states for Fermions, and Chrial Luutinger liquids to describe the low energy excitations of the fractional quantum Hall effect (foçr the end of the project).

References :

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Candidacy file/Dossier de candidature:

Preferably a minimal average of 14/20 for the M1 (and available M2 grades) is required. Candidates should have a solid background in Quantum Mechanics and Quantum Statistical Mechanics, Many Body Formalism and some research experience at the Master's level.

The candidacy file should contain : **a)** A letter of motivation and a short CV; **b)** Two reference letters, one from the head of the Master 2 (responsable de formation de Master 2), and another from the advisor who tutored the M1 internship (M2 if applicable); **c)** Grade transcripts with ranking if possible (relevé de notes) from the L1-L2-L3 and M1, and available grades for the M2. The file should be sent to **thierry.martin@cpt.univ-mrs.fr** before March 15th 2026. Note that further documents (such as high school diploma) are required by the Doctoral School. Candidates are welcome to contact me to arrange for a visioconference in order to ask questions.

For students who belong to the Ecole Normale Supérieure system, no further info is necessary.

For students who are not financed by the ENS, your candidacy will be transmitted to the Ecole Doctorale Physique et Sciences de la Matière ED 352 of Aix Marseille Université. Note that further documents (such as high school diploma) are required by the Doctoral School candidacy.
(see the website https://ecole-doctorale-352.univ-amu.fr/en).