Dynamical properties of the Hopfield model through bitwise arithmetic

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The Hopfield model provided the first simple yet powerful framework for understanding networks of interconnected neurons [1]. The model stores and retrieves memories, and its introduction started a mathematical synergy between neuroscience and computation, and laid conceptual foundations for modern research on neural networks.

The model is based on an energy function similar to that of spin=glass models, and it can be simulated with the well-known Metropolis algorithm. While the equilibrium (static) properties of the model are well known, much less is known about the dynamics that lead to equilibrium.

In this internship, we will focus on the effective dynamics of the Metropolis algorithm—or its variants—for the Hopfield model.

The Metropolis algorithm becomes computationally demanding when one attempts to sample a large number of configurations. This situation occurs, for example, when looking for rare samples in the dynamics, or sampling a large number of initial conditions. In this internship, we propose to design and implement a method to increase the computational yield of the algorithm, which leverages the boolean representation of numbers as they are stored in a computer [2].

This method allows for simulating simultaneously multiple copies of the system, which share the same random number used to perform the dynamical steps. Given that the random-number generation is the bottleneck of the simulation, this parallel algorithm yields a **significant computational gain**.

The student will be tasked with the numerical implementation of this parallel Metropolis algorithm, and will explore how the system converges to one of its metastable states by starting from different initial conditions, with particular emphasis on the **equilibration of the Monte Carlo dynamics**.

The student will acquire valuable interdisciplinary skills, such as proficiency in C++, and getting familiar models for neural networks. The internship will mainly take place at UMR 168, Institut Curie, and occasionally at Gulliver Laboratory, UMR 7083, ESPCI, Paris. For further information, please contact us at michele.castellana@curie.fr and david.lacoste@espci.fr .

References

[1] J. J. Hopfield. Neural networks and physical systems with emergent collective computational properties. *Proc. Natl. Acad. Sci. U.S.A.*, 79:2554–2558, 1982.

[2] Matteo Palassini and Sergio Caracciolo. Monte Carlo Simulation of the Three-dimensional Ising Spin Glass, September 2005.