Master 2 thesis project 2026

Finite-temperature properties of the toric code model in 3D and 4D

The toric code model is the simplest model featuring quantum topological order, i.e. long-range entanglement, in its ground state [Kitaev03]. Consequences are that the ground state has a topological degeneracy that depends on the genus and that excitations above the gap are fractionalized and known as anyons. It was originally defined in two space dimensions (2D). The 2D model is known to loose topological order at any finite temperature in the thermodynamic limit. The reason is the thermal proliferation of point-like topological defects (anyons), similarly to domain walls in the 1D classical Ising model.

A 3D version exists, with point-like electric excitations but loop-like magnetic excitations, that has a finite-temperature phase transition. However, even below the phase transition, long-range entanglement is not present at any finite temperature in this model. Only the 4D version is known to exhibit true topological order at small but finite temperature, see e.g. [Weinstein19]. Recently a variation of the 3D toric code model, known as the 3D fermionic toric code, was proposed that features true long-range entanglement at finite temperature [Zhou25].

In this 3-months internship starting in April 2026, we will explore the physics of topological order at finite temperature [Hastings11] working on the examples of the 3D toric code [Castelnovo08] and the 3D fermionic toric code [Zhou25]. In particular, we will try to understand the notion of true thermal topological order by contrasting the two models. As a follow up, we could also study the bosonic toric code model in 4D, that is expected to feature self-correction at finite temperature [Alicki10].

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