## Master 2 thesis project 2026

## Finite-temperature properties of fermionic topological order

"Topological order" is a concept proposed in the 90's by X.-G. Wen to describe the special kind of order at play in the fractional quantum Hall effect. For gapped two-dimensional (2D) systems, its hallmarks are: a ground-state degeneracy that depends on the genus of the real-space manifold on which the system resides (whether a sphere or a torus) and fractional excitations above the gap that are usually called anyons (as their most striking property is that of exotic quantum statistics). The simplest model of topological order is the famous toric code [Kitaev03]. It is a very simple example of bosonic topological order. Other exactly-solvable models of bosonic topological order are the Kitaev quantum double [Kitaev03] and the string nets of Levin and Wen [Levin05].

During a recent PhD thesis [Ritz-Zwilling24], we have computed the exact partition function of all these models (based on Hamiltonians that are sums of local commuting projectors) and found that it is almost that of two decoupled one-dimensional (1D) Potts models in which the number of "colors" or "states" q equals the total quantum dimension of the anyons. This "dimensional reduction" from a 2D quantum to a 1D classical model is also a subject of current interest.

We now wish to extend this work to another type of topological order, namely fermionic topological order. The main difference is that instead of starting from a lattice model in which the microscopic degrees of freedom are bosonic (such as localized two-level systems in the toric code), one starts with fermionic microscopic dofs. A simple example is the fermionic toric code [Gu14]. Generalizations can be obtained via fermionic string-nets [Gu15].

In this 3-months internship starting in April 2026, we will explore the physics of fermionic topological order. We will get familiar with the fermionic toric code, compute the degeneracy of its energy levels and from there obtain the partition function and study other finite-temperature properties such as entanglement entropy (or mutual information) and Wegner-Wilson loops. This will require getting familiar with mathematical concepts such as tensor categories.

## References:

[Kitaev03] A. Kitaev, arXiv:quant-phys/9707021 (Ann Phys 2003)

[Levin05] M. Levin and X.-G. Wen, arXiv:cond-mat/0404617 (Phys Rev B 2005)

[Ritz-Zwilling24] A. Ritz-Zwilling, PhD thesis Sorbonne Université 2024 https://dumas.ccsd.cnrs.fr/THESES-SU/tel-04824229v1

See also A. Ritz-Zwilling, B. Douçot, S.H. Simon, J. Vidal, J.-N. Fuchs, arXiv:2509.10876

[Gu14] Z.-C. Gu, Z. Wang and X.-G. Wen, arXiv:1309.7032 (Phys Rev B 2014)

[Gu15] Z.-C. Gu, Z. Wang and X.-G. Wen, arXiv:1010.1517 (Phys Rev B 2015)

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