Master 2 thesis project 2026

Finite-temperature properties of 2D fermionic topological order

"Topological order" is a concept proposed in the 90's by X.-G. Wen to describe the special kind of order at play in the fractional quantum Hall effect. For gapped two-dimensional (2D) systems, its hallmarks are: a ground-state degeneracy that depends on the genus of the real-space manifold on which the system resides (whether a sphere or a torus) and fractional excitations above the gap that are usually called anyons (as their most striking property is that of exotic quantum statistics). At zero temperature, topological order is characterized by the presence of long-range entanglement in the ground-state wavefunction.

The simplest model of topological order is the famous toric code [Kitaev03]. It is a very simple example of *bosonic* topological order, i.e. a model in which the microscopic degree of freedom are local and bosonic. Other exactly-solvable models of bosonic topological order are the Kitaev quantum double [Kitaev03] and the string nets of Levin and Wen [Levin05].

During a recent PhD thesis [Ritz-Zwilling24], we have computed the exact partition function of all these models featuring bosonic topological order (based on Hamiltonians that are sums of local commuting projectors) and found that it is almost that of two decoupled one-dimensional (1D) Potts models in which the number of "colors" or "states" q equals the total quantum dimension of the anyons. This "dimensional reduction" from a 2D quantum to a 1D classical model is also a subject of current interest (cf. higher-form symmetries).

We now wish to extend this work to another type of topological order, namely 2D *fermionic* topological order. The main difference is that instead of starting from a lattice model in which the microscopic degrees of freedom are bosonic and local (such as localized two-level systems in the toric code), one starts with fermionic microscopic dofs. A simple example is the 2D fermionic toric code [Gu14]. Generalizations can be obtained via fermionic string-nets [Gu15].

In this 3-months internship starting in April 2026, we will explore the physics of 2D fermionic topological order. We will get familiar with the 2D fermionic toric code, compute the degeneracy of its energy levels and from there obtain the partition function and study other finite-temperature properties such as entanglement entropy (or mutual information) and Wegner-Wilson loops. This will require getting familiar with mathematical concepts such as projective super fusion categories (i.e. Z_2-graded fusion categories).

References:

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[Gu14] Z.-C. Gu, Z. Wang and X.-G. Wen, arXiv:1309.7032 (Phys Rev B 2014)
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