Internship proposal: interaction of topological defects in a spatio-temporal crystal

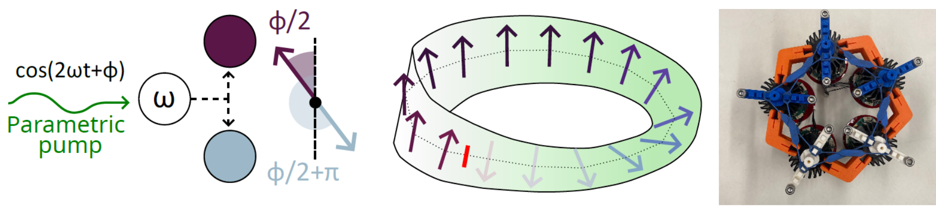

Figure 1: (Left) Two phase state of a parametric oscillator are represented by anti-aligned arrows. (Middle) Exciting a chain of parametric oscillators with a propagating wave corresponds to consider an effective spin chain on a Möbius strip. The latter must contain a defect (red dash) due to non-orientabilitly of the strip. (Right) Experimental realisation of such strip using mechanical oscillators.

A parametric oscillator is a simple example of dynamical system exhibiting two oscillating limit cycles. These two phase states can be used as analogic 0/1 memory storage. When coupled together, those oscillators can be used to mimic spin chains which energy is given by the Hamiltonian

$$H = -\sum J_{ij} s_i s_j \tag{1}$$

Finding the ground state of such Hamiltonian is generally a very hard task. However, arrays of coupled parametric oscillators can be shown to naturally relax towards the ground state of such Hamiltonian under certain conditions. Such analog computer is known as Ising machines with strong potential applications.

Recently, we examined the effect of asynchronous external forcing on such machines. Exciting an Ising machine with a propagating wave, we realized in practice a spatio-temporal crystal, i.e. a media which properties vary periodically both in space and time. Due to topological constrains, a phase dislocation between neighboring oscillators must appear somewhere. Interestingly, this dislocation is not fixed but rather moves along the chain. As a consequence, the system explores all the degenerated lowest energy states of the system. This new exploration mechanism of fundamental interest may help to improve standard Ising machines, that generally struggle to deal with frustration.

The goal of the present internship is to deepen our understanding of frustration propagation and interactions. In particular, how do several dislocations in a chain interact with each other ? We have seen numerically and experimentally spontaneous annihilation and creation of pairs of dislocations, but the underlying mechanism remains elusive. Other open questions include frustration propagation in 2D lattices, links with non-hermitian topological insulators and synchronization processes, that can be explored depending on the candidate's personal taste. The proposed topic combines theoretical, numerical and experimental approaches. All motivated students with strong background in physics (or mathematics) are welcome to apply. Basic knowledge in condensed matter/non-linear physics and Python programming are recommended, but not mandatory. The internship will take place in Laboratoire de Physique de l'ENS Lyon (site Monod).

Starting date: flexible around spring-summer 2026

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