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Flows and shapes of membrane with a protein inclusion

Many cellular functions rely on the ability of cells to alter their shape. At the cell-membrane level, shape changes are driven by forces which, regulated by specific proteins, alter the membrane curvature through structural alterations.

In this internship, we will study theoretically the fluid mechanics of a cell membrane—modelled as a two-dimensional fluid layer—including a trans-membrane protein. The main actors in this system are fluid flows along the membrane, and the membrane shape; the interplay between these two yields a **rich, unexplored physics**.

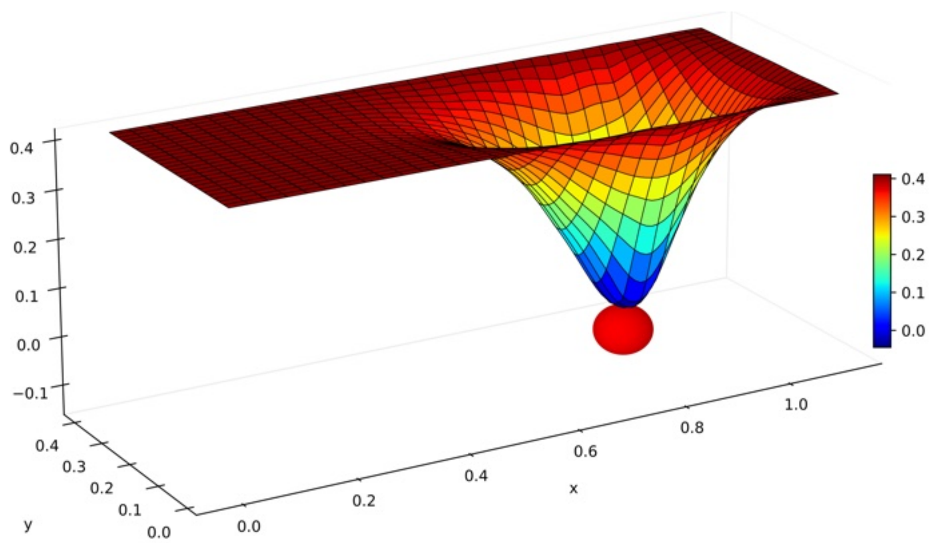

Figure 1: Steady state of a membrane in the absence of flows. The membrane (colored surface) is pulled downwards by the trans-membrane protein (red bead), and its shape is corresponds to a minimum-energy profile, obtained from a finite-element numerical solution.

In the absence of flows, the membrane is subjected to two forces: The Laplace pressure—which stems from the surface tension—and the bending rigidity; the membrane shape is then determined by a balance between these two forces, see Fig. 1.

The presence of flows in the membrane fluid alters this picture, see Fig. 2: in addition to the Laplace pressure and bending rigidity, viscous forces stemming from the flow alter the membrane shape. This intertwinement between flows and shape may reveal novel, physical features which we plan to study, such as:

- The transition between a 'laminar', time-independent steady state as in Fig. 2, to the onset of a turbulent,

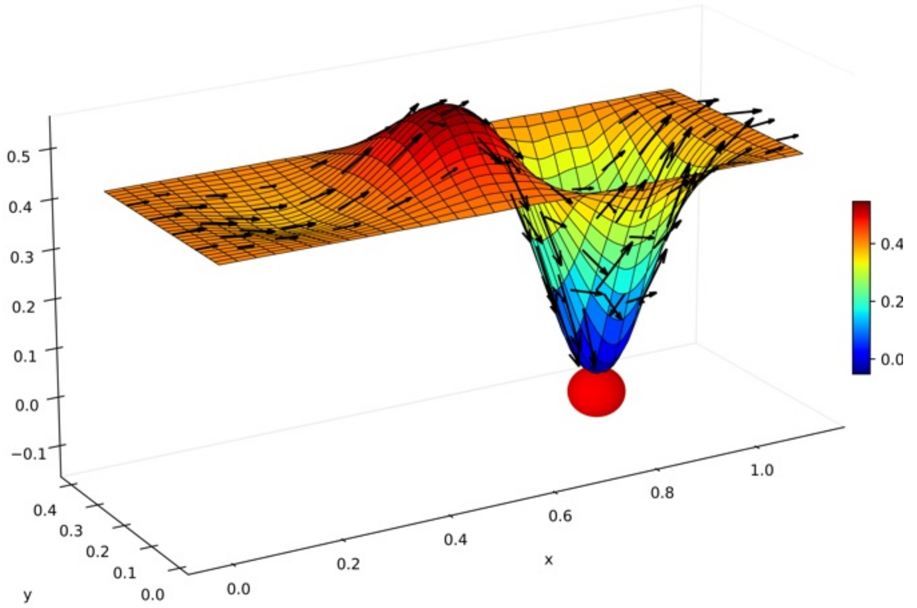

Figure 2: Steady state for a in the presence of flows (arrows), which alter the membrane shape, cf. Fig. 2.

potentially chaotic one for large Reynolds numbers, involving a **rich, coupled dynamics of flows and shape**.

- The nontrivial geometric structure of the fluid implies that its dynamical equations are strongly nonlinear, even in at zero Reynolds number [1]. This system may thus exhibit a **behavior typical of high-Reynolds-numbers even at low Reynolds numbers**; this would constitute a novel, emergent physical feature.
- Because of their scale invariance, the equations governing the fluid layer describe multiple phsical systems across a wide range of different viscosities, sizes and velocities, to which our results would then apply. These systems range from a few-μm thick biological membranez [2], which lie in the low-Reynolds-number regime, to macroscopic fluids in a fully turbulent state.

The student will be tasked with the numerical solution, with the finite-element method [3], of the partial differential equations which describe the fluid flow on a curved geometry [1], and with the analysis and development of the results.

The strong points of this internship are:

- **Scientific publications will be aimed at the best scientific journals**, both in physics and at the boundary between physics and biology. In fact, although this study is of a fundamental and theoretical nature, the innovative, multi-disciplinary and multi-scale character of this research will yield a broad impact of its results.
- **The student will acquire valuable skills**, such as proficiency in Python, learning and mastering of the finite-element method, and familiarity with the concepts of differential geometry, which will be highly beneficial for future endeavors.
- The lively scientific environment at UMR 168, Institut Curie will allow the student to find and establish new collaborations, in particular with experimentalists, to push forward and test the model precisions.
- The cross-disciplinary character of this internship, bridging between
  - theoretical physics (fluid dynamics),
  - differential geometry (fluid shape),
  - experimental physics and biology as for the experimental applications,

  will offer numerous directions for future developents.

For further information, please contact me.

[1] M. Arroyo and A. DeSimone. Relaxation dynamics of fluid membranes. *Phys. Rev. E*, 79(3):031915, March 2009.

[2] F. Quemeneur, J. K. Sigurdsson, M. Renner, P. J. Atzberger, P. Bassereau, and D. Lacoste. Shape matters in protein mobility within membranes. *Proc. Natl. Acad. Sci. U.S.A.*, 111(14):5083, April 2014.

[3] J. N. Reddy. *An introduction to the finite element method*. McGraw-Hill, Boston, 2006.