# PhD Training Proposal: Probes of anyonic statistics in fractional quantum Hall systems

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## SCIENTIFIC BACKGROUND

Fractional quantum Hall (FQH) systems provide one of the most remarkable realizations of strongly correlated quantum matter. In these two-dimensional electron gases, quasiparticles carry a fractional charge $e^*$ and obey exchange statistics interpolating between bosons and fermions, giving rise to *anyons*. For the simplest Laughlin states with filling $\nu = 1/(2k+1)$, one finds $e^* = \nu e$ and a statistical phase $\theta = \nu\pi$. More complex series, such as the Jain sequence, feature multiple quasiparticle species with distinct charges and exchange phases [1], whose identification has long relied on transport measurements through quantum point contacts (QPCs) embedded in Hall bars.

The first quantitative determinations of fractional charge were achieved via Poissonian DC shot-noise measurements [2], confirming Laughlin's predictions. The *Unifying Nonequilibrium Perturbative Theory* (UNEPT) [3] has since provided a significant theoretical advance, extending the validity of the Poissonian fluctuation–dissipation theorem (FDT) beyond bipartite and inversion-symmetric systems, even in regimes where Tomonaga–Luttinger liquid (TLL) models fail. UNEPT also offers robust, model-independent schemes for extracting the fractional charge $e^*$ from dynamical observables such as photoassisted current and finite-frequency noise [3], successfully implemented in experiments at complex filling factors [4, 5], which reveal deviations from standard TLL behavior.

In contrast, the direct observation of anyonic statistics remains a major challenge despite remarkable experimental progress. Existing methods probe braiding either in space or in time. Spatial braiding has been investigated using interferometric geometries [6–8], but these measurements are often hindered by strong Coulomb interactions and device-specific effects [9, 10]. Temporal braiding has been addressed through current cross-correlations in Hanbury-Brown-Twiss [11–13] and "anyon collider" geometries [14], realized in pioneering experiments by G. Fève [15]. However, these approaches cannot disentangle the braiding phase $\theta$ from nonuniversal parameters such as the scaling dimension $\delta$ or plasmonic-mode renormalization, and discrepancies with theoretical predictions have been reported [16, 17]. Time-resolved Hong-Ou-Mandel interferometry [18, 19], based on pulse shaping by an AC drive [20], has further advanced the field, though its interpretation remains debated [21].

To overcome these limitations, UNEPT has recently been extended to tackle fractional statistics without relying on TLL assumptions. It provides original, experimentally accessible methods to probe the anyonic braiding phase $\theta$ through measurable transport quantities such as finite-frequency noise and admittance [3, 22, 23]. These developments open the way for systematic exploration of anyonic statistics in nonequilibrium and driven regimes.

## OBJECTIVES

The goal of this project is to develop theoretical tools and experimental protocols for probing anyonic statistics in both time and frequency domains within the UNEPT framework. It builds on three recent studies [21–23], two co-authored with postdoctoral researcher Aleksander Latyshev (formerly at the University of Geneva with E. Sukhorukov, where he has co-supervised a PhD student), who will contribute actively to the supervision.

**(1) Extension of the braiding FDT to finite DC and AC drives.** Recent work introduced a *braiding fluctuation–dissipation theorem* (braiding FDT) linking measurable quantities—such as DC noise, conductance, and admittance—to the anyonic braiding phase $\theta$, even under nonequilibrium conditions. A first objective is to extend this relation to finite DC and AC biases, providing a unified framework for experimental investigation. Within thermal regimes, this theory reveals that $\theta$ is related to the scaling dimension $\delta$, suggesting that the time-braiding phase may not be strictly universal but instead reflect edge-state dynamics. The PhD candidate will learn to derive these relations analytically and delimit the class of models consistent with time-domain braiding.

**(2) Extension to interferometers and space–time braiding.** UNEPT has been recently generalized to describe interferometers under AC drives, offering a unified treatment of photoassisted transport and coherence in Mach-

Zehnder and Fabry-Pérot geometries. Since the time-braiding phase depends on nonuniversal effects, exploring the corresponding space-braiding phase through interferometers provides a promising route to identify universal topological signatures. The student will analyze this duality to build a consistent description of anyonic statistics across temporal and spatial domains.

## WORK PLAN AND TRAINING

The training combines analytical modeling, theoretical development, and strong collaboration with experimental teams at ENS Paris (G. Fève), C2N Saclay (F. Pierre), and ENS Lyon (P. Degiovanni). The main tasks include:

- Generalize the braiding FDT to nonequilibrium configurations with multiple QPCs, temperature gradients, and dephasing.
- Extend UNEPT to interferometers under AC drives and establish the connection between temporal and spatial braiding.
- Identify experimental observables—phase-resolved admittance, finite-frequency noise-allowing direct extraction of $\theta$ and $\delta$.
- Collaborate with experimental groups to design feasible measurement protocols within current FQH and circuit-QED platforms.

The candidate will be trained in advanced non-equilibrium quantum transport, field-theoretical methods, and FQHE edge-state physics, gaining direct exposure to experimental practice through ongoing collaborations.

## EXPECTED OUTCOMES AND PERSPECTIVES

The project will establish a unified framework to probe anyonic statistics via both temporal and spatial interference signatures. By clarifying the relation between the braiding phase $\theta$ and the scaling dimension $\delta$, it will distinguish universal topological effects from nonuniversal interaction-driven dynamics. Beyond the FQH regime, these methods can be extended to other strongly correlated or topological systems, including hybrid superconducting structures hosting Majorana or parafermionic excitations, and engineered quantum circuits emulating one-dimensional quantum fluids. Overall, this work will consolidate UNEPT as a versatile tool for nonequilibrium quantum transport and dynamical braiding phenomena, strengthening the theoretical–experimental synergy at LPS Orsay and partner institutions.


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