INTERNSHIP PROPOSAL

(One page maximum)

Laboratory name: Thales Alenia Space Cannes
Internship director'surname: N. Allegra, S. Dartois (Université Bordeaux) and A. Tanasa (Université Bordeaux)
e-mail: nicolas.allegra@thalesaleniaspace.com stephane.dartois@u-bordeaux.fr ntanasa@u-bordeaux.fr Phone number: 0033.5.40.22.50
Web page: https://stephanedartois.com/ and https://www.labri.fr/perso/atanasa/
Internship location: Thales Alenia Space Cannes
Thesis possibility after internship: YES
Funding: NO

Efficient Pauli String Decomposition of Random Matrices for Quantum Algorithms

Physical context: The partial differential equations that model propagation in random media (such as the atmosphere) involve random operators, which we propose to approximate efficiently using quantum computing methods and/or tensor network methods (which belong to the class of so-called "quantum-inspired" methods).

Thus, the decomposition of a square matrix into a sum of Pauli strings is a standard pre-processing step required to implement many quantum algorithms (which are viewed as quantum circuits made of various types of quantum gates - CNOT Hadamard, etc.). However, such a Pauli decomposition demands significant computational resources in the case of large matrices.

The **topic of the internship** revolves around the approximation of random matrices using such Pauli decompositions, having as a later objective the topic of applications to optics within turbulent media.

From a mathematical point of view, it is known that any unitary matrix of size N can be written as a decomposition into tensor products of Pauli matrices on a Hilbert space of size $n = log(N)$ qubits. However, in general, the number of terms grows exponentially with n, and each coefficient is also computed in exponential time.

In simple particular cases, such as certain types of diagonal matrices, the exact form of the decomposition can be exactly computed. Thus, the number of terms in the series can remain finite as the size n grows, allowing thus the Pauli decomposition to be expressed with a finite number of CNOT gates and z-rotations.

This can be extended to diagonal, yet random matrices. In this particular case, it becomes possible to compute the number of terms required on average to achieve a given level of approximation.

The **objective of the internship** is to tackle the case of non-diagonal matrices, initially through **numerical experimentation**. To this end, we propose using certain existing methods that efficiently decompose the matrix under study, thereby testing the required number of terms in the decomposition.

Please, indicate which speciality(ies) seem(s) to be more adapted to the subject:

Condensed Matter Physics: NO Soft Matter and Biological Physics: NO
Quantum Physics: YES Theoretical Physics: YES